Trigonometric Ratios

For a **right angle triangle**, the relationship between lengths of sides and angles is described using the trigonometric ratios.

The 3 **primary trigonometric ratios** are: sine (sin), cosine (cos) and tangent (tan).

For a given angle, A, the primary trig ratios are defined as follows:

$$\text{Sin(A)} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cos(A)} = \frac{\text{adjancent}}{\text{hypotenuse}}$$

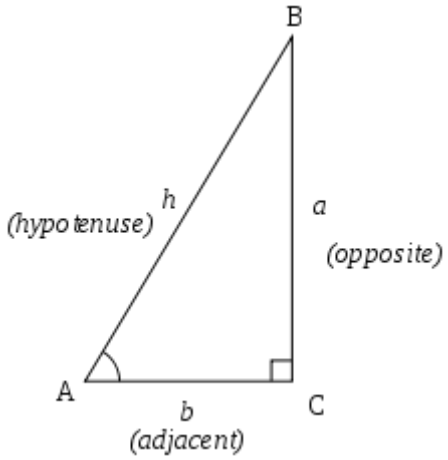
$$\text{Tan(A)} = \frac{\text{opposite}}{\text{adjacent}}$$

Note: “opposite” refers to the side length **opposite** from angle A.

“Hypotenuse” is *always* the side opposite the 90º angle.

“Adjacent” refers to the side length **adjacent** to (i.e. beside) angle A.

See Figure 1. below.



Note: a useful way to remember the primary trig ratios is the acronym **SOH CAH TOA**.

In addition to the primary trigonometric ratios, there are 3 **reciprocal trigonometric ratios**: cosecant (csc), secant (sec) and cotangent (cot).

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For a given angle, A, the reciprocal trig ratios are defined as follows:

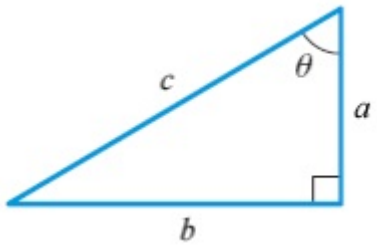
$$\text{Csc}(A) = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{Sec}(A) = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{Cot}(A) = \frac{\text{adjacent}}{\text{opposite}}$$

Example 1:

Find the exact values of the six trigonometric ratios for angle θ in the triangle below if a = 3, b = 4, and c = 5.



Solution:

$\sin\theta = \frac{b}{c} = \frac{4}{5} = 0.8$

$\cos\theta = \frac{a}{c} = \frac{3}{5} = 0.6$

$\tan\theta = \frac{b}{a} = \frac{4}{3}$

$\csc\theta = \frac{c}{b} = \frac{5}{4} = 1.25$

$\sec\theta = \frac{c}{a} = \frac{5}{3}$

$\cot\theta = \frac{a}{b} = \frac{3}{4} = 0.75$

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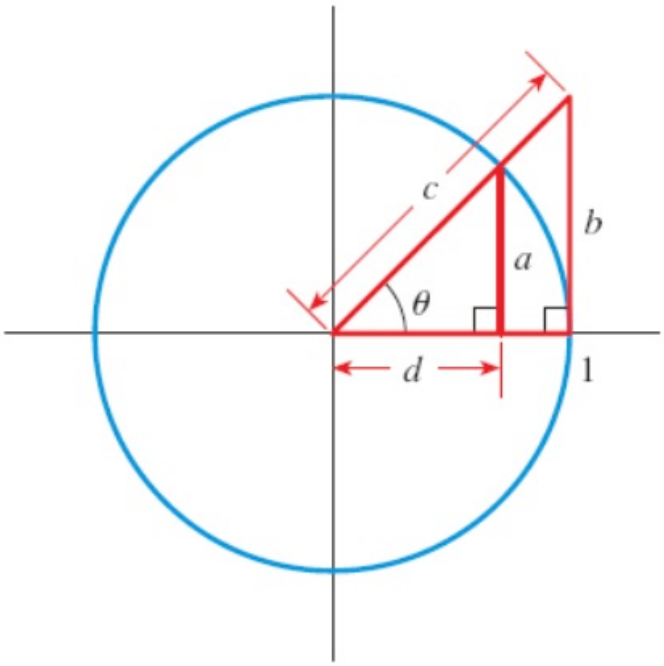
Example 2:

Express the lengths a, b, c and d in terms of trigonometric ratios of θ. Refer to the figure below.



Solution:	Explanation:
a = sinθ	Side a is located opposite angle θ in the smaller triangle. The hypotenuse of the smaller triangle is equal to 1 (since it's the radius of the circle). Thus, $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{a}{1}$, which is simplified to a = sinθ.
b = tanθ	Side b is located opposite angle θ in the larger triangle. The adjacent side of the larger triangle is equal to 1 (since it's the radius of the circle). Thus, $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{b}{1}$, which is simplified to b = tanθ.
c = secθ	Side c is the hypotenuse of the larger triangle. The adjacent side is the radius of the circle and equal to 1. Thus, $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{c}{1}$, which is simplified to c = secθ.
d = cosθ	Side d is located adjacent to angle θ in the smaller triangle. The hypotenuse of the smaller triangle is equal to 1 (since it's the radius of the circle). Thus, $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{d}{1}$, which is simplified to d = cosθ

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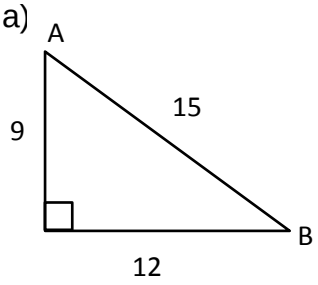
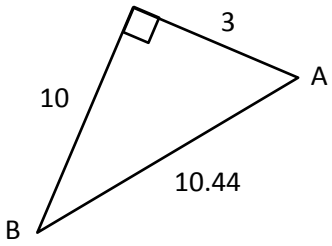
Practice Questions:

1. State the six trigonometric ratios for angles A and B in the triangles below. For part a, state the ratios as exact fractions. For part b, state the ratios as decimals rounded to the nearest hundredth.

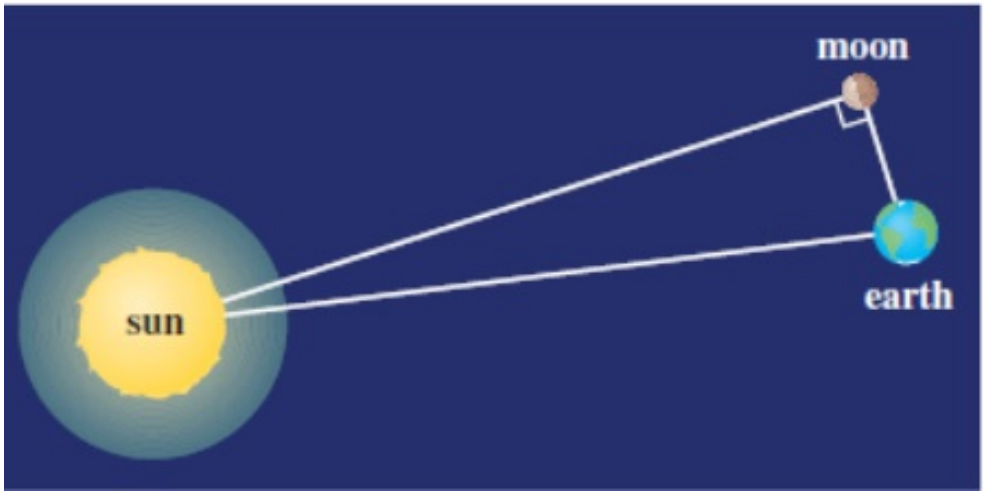
a)



b)



2. When the moon is exactly half full, the earth, moon, and sun form a right angle (see the figure below). At that time the angle formed by the sun, earth, and moon (using the earth as its vertex) is measured to be 89.85°. If the distance from the earth to the moon is 240,000 miles, estimate the distance from the earth to the sun. (Round your answer to one decimal place.)



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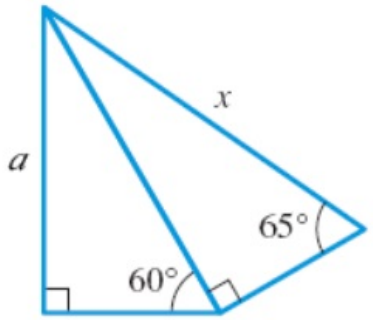
3. Find x in the figure below. Assume a = 65 meters. Round the answer to two decimal places



Answers:

1. a)

sin A = $\frac{4}{5}$	sin B = $\frac{3}{5}$
cos A = $\frac{3}{5}$	cos B = $\frac{4}{5}$
tan A = $\frac{4}{3}$	tan B = $\frac{3}{4}$
csc A = $\frac{5}{4}$	csc B = $\frac{5}{3}$
sec A = $\frac{5}{3}$	sec B = $\frac{5}{4}$
cot A = $\frac{3}{4}$	cot B = $\frac{4}{3}$

b)

sin A = 0.96	sin B = 0.29
cos A = 0.29	cos B = 0.96
tan A = 3.33	tan B = 0.3
csc A = 1.04	csc B = 3.48
sec A = 3.48	sec B =1.04
cot A = 0.30	cot B = 3.33

2. 91,673.35 miles

3. 82.81 meters