Percentage Change

It is often useful to know by how much an item's values or properties change over time. The **percentage change** measures an item's change in value relative to its original value. It has numerous applications in real-life scenarios and most fields of study.

$$\% \ change = \frac{new \ value - old \ value}{old \ value} \times 100\%$$

Note that % change can also be called % increase, % decrease, % error, and many more names depending on what it is measuring.

Example 1. At KFC, the $2 Toonie Tuesday special had a price increase to $2.39. Calculate the % increase in its price.

$$\% \ change = \frac{\$2.39 - \$2.00}{\$2.00} \times 100\% = \frac{\$0.39}{\$2.00} \times 100\% = 0.195 \times 100\% = 19.5\%$$

Example 2. An air turbine decreases its rotational speed from 5 rpm (rotations per minute) to 3 rpm. Calculate the % decrease in its speed.

$$\% \ change = \frac{3rpm - 5rpm}{5rpm} \times 100\% = \frac{-2rpm}{5rpm} \times 100\% = -0.40 \times 100\% = -40\%$$

Notice that a <u>positive</u> percentage change denotes a **percentage increase**, whereas a <u>negative</u> percentage change denotes a **percentage decrease**.

For example, a 20% decrease is the same as a −20% change.

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Percent Error

The **percent error** is used in many scientific applications. In labs, it measures how accurate an experiment is, compared to the expected (often numerical) outcome.

$$\% \, error = \frac{experimental - theoretical}{theoretical} \times 100\% = \frac{actual - expected}{expected} \times 100\%$$

Example 3. Becky dropped a ball off of a balcony and calculated that it would take 2.0 seconds to hit the ground. In actuality, the ball took 2.12 seconds to hit the ground. Find the % error in her time.

$$\% \, error = \frac{2.12s - 2.00s}{2.00s} \times 100\% = \frac{0.12s}{2.00s} \times 100\% = 0.06 \times 100\% = 6.00\%$$

Example 4. Andy shot a flare, calculating that it would reach a maximum height of 400m when it actually reached a height of 450m. Find the % error in his calculation.

$$\% \, error = \frac{450m - 400m}{400m} \times 100\% = \frac{50m}{400m} \times 100\% = 0.125 \times 100\% = 12.5\%$$

Example 5. Dylan estimated that it would take him 5 hours to drive to Ottawa from Toronto, but when he made the journey, it took him 4 hours. Find the % error in his estimation.

$$\% \, error = \frac{4 \, hours - 5 \, hours}{5 \, hours} \times 100\% = \frac{-1 \, hour}{5 \, hours} \times 100\% = -0.20 \times 100\% = -20\%$$

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More Difficult Problems Using Percent Change

Example 6. The price of a $10 000 ancient Ming Dynasty Chinese urn circa 1473 saw a recent price increase of 12%. Calculate its new price.

$$new\ price = 112\%\ of\ \$10,000 = 1.12 \times \$10,000 = \$11,200$$

Example 7. The price of a $5.30 vintage iPod Touch circa 1300 BCE as advertised on eBay saw a <u>drop</u> in retail value of 2%. What was its original price?

$$original\ price = \frac{\$5.30}{98\%} = \frac{\$5.30}{0.98} = \$5.41$$

Example 8. An often-used approximation of π is 3.14. Use a calculator to find the % error in using 3.14 instead of the exact value of π when calculating the area of a circle. Note: The area of a circle is $A = \pi r^2$.

The actual area of the circle is $A = \pi r^2$.
The expected area when using the approximation is $A = 3.14r^2$.

$$\%\ error = \frac{\pi r^2 - 3.14r^2}{3.14r^2} \times 100\% = \frac{\pi - 3.14}{3.14} \times 100\% = 0.0005 \times 100\% = 0.05\%$$

Example 9. A surveyor estimated the length of a stretch of road to be 800m. It was actually 0.9km. What was the % error on his estimation?

$$\%\ error = \frac{0.9km - 800m}{800m} \times 100\% = \frac{900m - 800m}{800m} \times 100\%$$
$$= \frac{100m}{800m} \times 100\% = 0.125 \times 100\% = 12.5\%$$

Notice that you have to convert everything to the same units before completing the calculation. You get the same answer whether you convert all the units to meters or kilometers.

$$\%\ error = \frac{0.9km - 800m}{800m} \times 100\% = \frac{0.9km - 0.8km}{0.8km} \times 100\%$$
$$= \frac{0.1km}{0.8km} \times 100\% = 0.125 \times 100\% = 12.5\%$$

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Questions and Exercises

Round your answers to two decimals places.

1. Sasha timed how long she took to complete two laps of the track. Her first trip took 36.5s. Her second trip took 35.8s. Calculate the % decrease in the time she took to complete her second lap.

2. Michael swung his arms in circles for 2 minutes. Over the first minute, he averaged 80 rpm (rotations per minute). Over the second minute, he averaged 85 rpm. Calculate the % increase in his rpm from the first to the second minute.

3. Elisa went on a shopping spree last week. Recently she got quite upset because an item that she purchased for $500 last week is available at a price of $350 this week. Calculate the percent decrease in the price of the item.

4. Arpit combined $CuCl_{2(s)}$ and $NaOH_{(aq)}$ and calculated that he would produce 3 grams of $NaCl_{(s)}$. When he measured it, there were 2.8 grams. Find the % error.

5. A car is moving along the road at 70 km/h.
   a. As the car enters the highway, its speed increases by 40%. What is its new speed?
   b. When the car leaves the highway, its speed decreases to 50 km/h. What is its % decrease in speed compared to the car's speed on the highway?

6. Carbon dating is an effective way to measure the age of organic materials. By measuring the presence of the C^{14} isotope, a suitable estimate can be found for how old/authentic an artifact is. A fossil dug up at an archaeological site is examined and it is found that its mass in C^{14} has decayed by 99.7% to 0.800g. What was its original mass?

7. An experiment with a strain of bacteria's response to radiation shrinks its population by 15% to a size of 300. What was its original population?

8. Ruslan grabs a bunch of bananas at the local grocery store and estimates that it weighs about 1200 g. When he goes to the register, the cashier weighs it and finds that it is 1.0 kg. What is the % error in his estimate?

Percentage Change

Solutions

1. 1.92%

2. 6.25%

3. 30%

4. -6.67%

5. a. 98 km/h   b. 48.98%

6. 267g

7. 353

8. -20%