Solving Linear Equations in One Variable

A linear equation is an algebraic equation with a degree of 1. This means that the highest exponent on any variable in the equation is 1.

A **linear equation in one variable** can be written in the form $ax + b = c$, where a, b, and c are real numbers.

General *guidelines* for solving linear equations in one variable:

1. Simplify anything inside brackets.
2. Get rid of any brackets using the distributive property: a(b+c) = ab + ac
3. Collect like terms.
4. Isolate the unknown variable by moving all other terms to the other side of the equation. To move a term across the equal sign, do the *opposite* operation on the other side. (Addition → subtraction; subtraction → addition; multiplication → division; division → multiplication).
5. Linear equations of the form $\frac{a}{b} = \frac{c}{d}$ can be first simplified using cross multiplication. $\frac{a}{b} = \frac{c}{d}$ becomes $ad = bc$

Note: The above are *guidelines* only and are NOT a step-by-step guide to solving linear equations in one variable. Different equations will require different techniques for solving. In many cases, there is more than one way to solve a linear equation.

Examples:

Solve for x.

Example 1: $2(x + 1) = 3(4 - x)$

$2(x + 1) = 3(4 - x)$	Expand the brackets.
$2x + 2 = 12 - 3x$	Move the x terms to the same side and the numbers to the other side.
$2x + 3x = 12 - 2$	Collect like terms.
$5x = 10$	Move 5 to the other side by dividing by 5 on the right side.
$x = \dfrac{10}{5}$	Do the division.
$x = 2$	

Example 2: $\frac{2}{3}x - \frac{1}{2}x = \frac{4}{3}$

$\frac{2}{3}x - \frac{4}{3} = \frac{1}{2}$	Move the like terms to the same side of the equation.
$\frac{2}{3}x - \frac{1}{2}x = \frac{4}{3}$	In order to subtract the x terms find a common denominator and re-write both terms as equivalent fractions with the same denominator.
$\frac{4}{6}x - \frac{3}{6}x = \frac{4}{3}$	Subtract like terms.
$\frac{1}{6}x = \frac{4}{3}$	Multiply x and $\frac{1}{6}$
$\frac{x}{6} = \frac{4}{3}$	Move 6 to the other side by multiplying by 6 on the right side.
$x = \frac{4}{3}(6)$	Do the multiplication.
$x = \frac{24}{3}$	Reduce the fraction.
$x = 8$	

Example 3: $\frac{3}{x} + 4 = 2$

$\frac{3}{x} + 4 = 2$	Move 4 to the right side of the equation.
$\frac{3}{x} = 2 - 4$	Subtract.
$\frac{3}{x} = -2$	Cross multiply.
$\frac{3}{x} = \frac{-2}{1}$	
$3 = -2x$	Move −2 to the right side of the equation by doing the opposite operation.
$x = -\frac{3}{2}$ or $x = -1\frac{1}{2}$	Leave your answer as an improper fraction or change it to a mixed number.

Example 4: $\frac{3x-1}{5} = 3x + 1$

$\dfrac{3x-1}{5} = 3x+1$ $\dfrac{3x-1}{5} = \dfrac{3x+1}{1}$	Cross multiply to get rid of fractions on both sides of the equation. (Note that 3x + 1 can be written as a fraction with a denominator of 1).
$3x-1 = 5(3x+1)$	Expand the brackets on the right side using the distributive property.
$3x-1 = 15x+5$	Move 15x to the left side of the equation. Move −1 to the right side of the equation.
$3x - 15x = 5 + 1$	Collect like terms.
$-12x = 6$	Move −12 to the right side of the equation by doing the opposite operation.
$x = -\dfrac{6}{12}$ $x = -\dfrac{1}{2}$	Simplify the fraction by reducing to lowest terms.

Example 5: $\frac{2x+1}{3} + \frac{1}{2} = 1 - \frac{x-3}{5}$

$\dfrac{2x+1}{3} + \dfrac{1}{2} = 1 - \dfrac{x-3}{5}$	To get rid of fractions on both sides of the equation, first find the lowest common multiple (LCM) of the denominators.
$30\left(\dfrac{2x+1}{3}\right) + 30\left(\dfrac{1}{2}\right) = 30(1) - 30\left(\dfrac{x-3}{5}\right)$	Multiply EVERY term in the equation by the LCM. Simplify each term. Fractions can be reduced.
$10(2x+1) + 15(1) = 30 - 6(x-3)$	Get rid of brackets by using the distributive property.
$20x + 10 + 15 = 30 - 6x + 18$	Move like terms to the same side of the equation.
$20x + 6x = 30 + 18 - 10 - 15$	Collect like terms.
$26x = 23$	Move 26 to the right side of the equation by doing the opposite operation.
$x = \dfrac{23}{26}$	

Practice Questions:

1. Solve for x.

   a) $3 - 4x = 8x + 3$

   b) $20x + 4x - 18 = 30 - 6x$

   c) $1 - (2x + 5) = -3x$

   d) $-24(10)x + 19x = 76x - (9x + 2)$

   e) $8(x - 3) - 2(x - 2) = 20$

   f) $x - 3 = 2(x + 5) + 2x + 2$

   g) $\frac{1}{2}(x - 6) + \frac{3}{5}(x + 10) = 24$

   h) $1 - \frac{x}{3} = 6$

   i) $\frac{x}{2} - \frac{3x}{4} = 1$

   j) $\frac{2x}{3} + \frac{7x}{6} = 5x + 30$

Answers:

1. a) x = 0

    b) x = 1.6

    c) x = 4

    d) $x = \frac{1}{144}$

    e) $x = 6\frac{2}{3}$

    f) x = −5

    g) $x = 19\frac{1}{11}$

    h) x = −15

    i) x = −4

    j) $x = -9\frac{9}{19}$