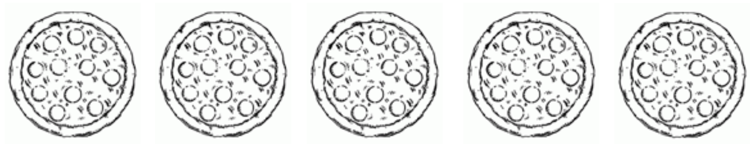
Introduction to Fractions

Part A – What is a Fraction?

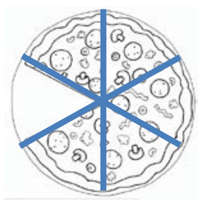
Count the number of whole pizzas. How many do we have?



1 2 3 4 5

We have **5 whole** pizzas.

Count the number of whole pizzas. How many do we have?



We have **less than one whole** pizza.

Sometimes we cannot count in whole numbers. In these cases we need to express quantities in parts. Everyday words such as slices, parts, sections, chunks, and pieces are used to express parts of a whole. In math, we use **fractions** to depict these values.

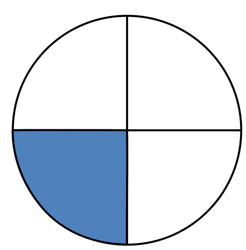
A **fraction** consists of two numbers separated by a bar in between them.

- The ***bottom*** number, called the **denominator,** is the total number of **equally divided** portions in one whole.
- The ***top*** number, called **the numerator,** is how many portions you have.
- The bar represents the operation of **division**.

Numerator → We have 1 out of the 4 portions.

$$\frac{1}{4}$$

← **Denominator** The whole is divided into four equal parts.



We always assume that every whole is divided into **equal** parts. Observe that Figure 1.1 has equally divided slices while Figure 1.2 does not.

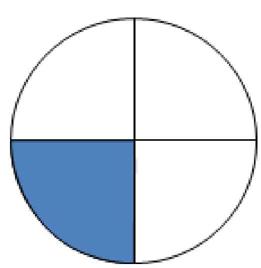


Figure 1.1

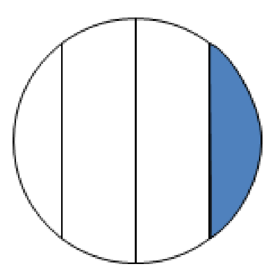


Figure 1.2

Introduction to Fractions

Part B – Creating Equivalent Fractions

Fractions can be written in many ways, all of which represent the same value. Fractions that represent the same value are called **equivalent fractions**.

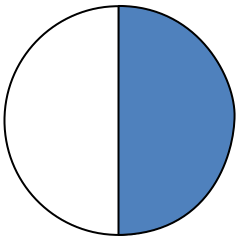


Figure 1.3

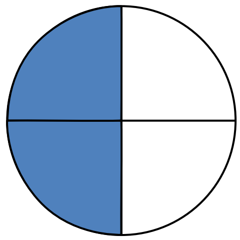


Figure 1.4

$$\frac{1}{2} = \frac{2}{4}$$

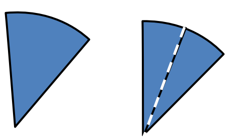
For example, in Figure 1.3 we sliced a whole pizza into two pieces. In figure 1.4 we sliced a whole pizza into four equal pieces.
Even though the pieces are different in size, they are equivalent fractions because they are both equally shaded.

To create equivalent fractions, **multiply** the **numerator** and **denominator** by the **same number**.

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$$

$\times 2$ $\times 3$ $\times 4$ $\times 5$

How does this work?



1) If we cut this piece in half, then we have 2 smaller pieces.

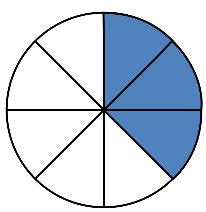


Figure 1.6

$$\frac{3 \xrightarrow{\times 2} 6}{8 \xrightarrow{\times 2} 16}$$

2) By multiplying the numerator and denominator by 2, we are cutting our 8 portions into 16 portions. Nonetheless, we still have the same amount of shaded area.

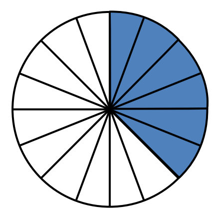


Figure 1.7

Example 1: Find the missing number in, $\frac{\ }{12} = \frac{3}{4}$ to create an equivalent fraction.

Remember, equivalent fractions are converted by multiplying the numerator and the denominator by the same number. Thus, our goal is to find that number.

$$\frac{\ }{12} = \frac{3}{4} \quad \times 3$$

$$\frac{9}{12} = \frac{3}{4} \quad \times 3$$

Introduction to Fractions

We learned that we can create equivalent fractions by **multiplying** the numerator and denominator by the same number. This process can create really big fractions. A second method to create equivalent fractions is to **divide** the numerator and denominator by its **greatest common factor.** (Note: this is only possible when a fraction can be reduced).

Note: The **greatest common factor** is the largest number that is a factor of both numbers.

Example 2: Find the Greatest Common Factor between 64 and 56

List all the factors of 64 and 56.

Factors of 64: 1, **2**, **4**, $\boxed{\mathbf{8}}$, 16, 32, 64
Factors of 56: 1, **2**, **4**, 7, $\boxed{\mathbf{8}}$, 14, 28,

Notice that 64 and 56 have 2, 4, and 8 as common factors, but we are looking for the greatest one. Thus, the GCF between 64 and 56 is 8.

Reducing/Simplifying Fractions

Step 1: Find the **Greatest Common Factor (GCF)** between the numerator and the denominator.
Step 2: Divide the numerator and the denominator by the GCF.
Step 3: If the GCF is equal to 1, then the fraction is already in its most simplified form.

Example 3: Simplify the following fraction, $\frac{28}{32}$.

First, let's find the GCF of 28 and 32.

Factors of 28: 1, 2, $\boxed{4}$, 7, 14, 28
Factors of 32: 1, 2, $\boxed{4}$, 8, 16, 32

The GCF of 28 and 32 is **4**. Divide the numerator and denominator by 4.

$$\frac{28}{32} \underset{\div 4}{\overset{\div 4}{=}} \frac{7}{8}$$

Example 4: Simplify the following fraction, $\frac{7}{9}$.

First, let's find the GCF of 7 and 9.

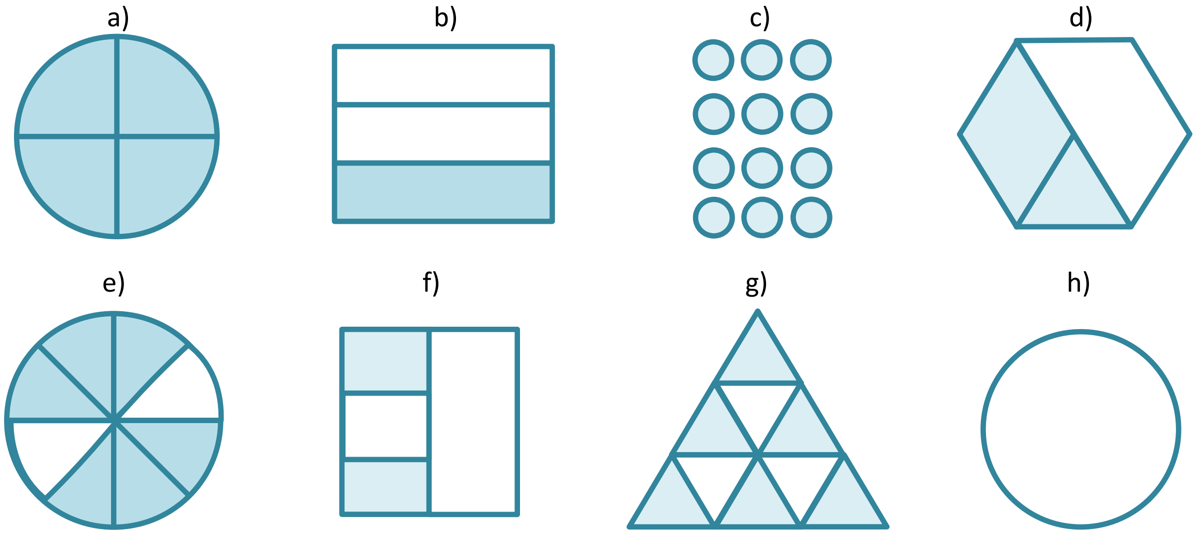
Factors of 7: 1, 7
Factors of 9: 1, 3, 9

Since the GCF of 7 and 9 is 1, $\frac{7}{9}$ is at its simplest form.

Introduction to Fractions

Exercises:

1. Express the following diagrams as fractions.



2. Draw a picture to express the following fractions.

a) $\frac{2}{5}$	b) $\frac{5}{6}$	c) $\frac{6}{12}$	d) $\frac{7}{8}$
e) $\frac{4}{6}$	f) $\frac{3}{10}$	g) $\frac{1}{3}$	h) $\frac{3}{2}$

3. Express the following statements as fractions.

   a) 45 minutes of an hour
   b) 2 liters filled out of 5 liters
   c) 3 laps of 4 laps
   d) 300 meters of one 1 kilometer
   e) 100 milliliters out of a cup (250 milliliters)

4. Find six equivalent fractions for the following.

a) $\frac{4}{7} = \frac{\quad}{14} = \frac{\quad}{21} = \frac{\quad}{28} = \frac{\quad}{35} = \frac{\quad}{42}$

b) $\frac{1}{3} = \frac{2}{\quad} = \frac{4}{\quad} = \frac{6}{\quad} = \frac{8}{\quad} = \frac{10}{\quad}$

5. Find the Greatest Common Factor between the pair of numbers.

a) $3, 6$ b) $4, 8$ c) $7, 10$ d) $1, 4$
e) $12, 18$ f) $15, 18$ g) $13, 39$ h) $24, 2$

6. Simplify the following fractions.

a) $\frac{15}{18}$ b) $\frac{18}{20}$ c) $\frac{14}{16}$ d) $\frac{35}{40}$
e) $\frac{10}{70}$ f) $\frac{20}{80}$ g) $\frac{21}{35}$ h) $\frac{9}{15}$
i) $\frac{8}{24}$ j) $\frac{5}{5}$ k) $\frac{4}{5}$ l) $\frac{9}{81}$

Solutions:

1. Express the following diagrams as fractions.

a) $\frac{4}{4} = 1\ \boldsymbol{whole}$ **b)** $\frac{1}{3}$ **c) 12 wholes** **d)** $\frac{3}{6} = \frac{1}{2}$
e) $\frac{6}{8}$ **f)** $\frac{2}{6}$ **g)** $\frac{6}{9}$ **h) 0**

2. Draw a picture to express the following fractions.

3. Express the following statements as fractions.

a) $\frac{45}{60}$ **b)** $\frac{2}{5}$ **c)** $\frac{3}{4}$ **d)** $\frac{300}{1000}$ **e)** $\frac{100}{250}$

4. Find six equivalent fractions for the following.

a) $\frac{4}{7} = \frac{\mathbf{8}}{14} = \frac{\mathbf{12}}{21} = \frac{\mathbf{16}}{28} = \frac{\mathbf{20}}{35} = \frac{\mathbf{24}}{42}$ **b)** $\frac{1}{3} = \frac{2}{\mathbf{6}} = \frac{4}{\mathbf{12}} = \frac{6}{\mathbf{18}} = \frac{8}{\mathbf{24}} = \frac{10}{\mathbf{30}}$

5. Find the Greatest Common Factor between the two pairs of number.

a) $3, 6 =$ **3** b) $4, 8 =$ **4** c) $7, 10 =$ **1** d) $1, 4 =$ **1**
e) $12, 18 =$ **6** f) $15, 18 =$ **3** g) $13, 39 =$ **13** h) $24, 2 =$ **2**

6. Simplify the following fractions.

a) $\frac{15}{18} = \frac{\mathbf{5}}{\mathbf{6}}$ b) $\frac{18}{20} = \frac{\mathbf{9}}{\mathbf{10}}$ c) $\frac{14}{16} = \frac{\mathbf{7}}{\mathbf{8}}$ d) $\frac{35}{40} = \frac{\mathbf{7}}{\mathbf{8}}$ e) $\frac{10}{70} = \frac{\mathbf{1}}{\mathbf{7}}$ f) $\frac{20}{80} = \frac{\mathbf{1}}{\mathbf{4}}$
g) $\frac{21}{35} = \frac{\mathbf{3}}{\mathbf{5}}$ h) $\frac{9}{15} = \frac{\mathbf{3}}{\mathbf{5}}$ i) $\frac{8}{24} = \frac{\mathbf{1}}{\mathbf{3}}$ j) $\frac{5}{5} =$ **1** whole k) $\frac{4}{5} = \frac{\mathbf{4}}{\mathbf{5}}$ l) $\frac{9}{81} = \frac{\mathbf{1}}{\mathbf{9}}$