Adding and Subtracting Proper Fractions

Just like our counting numbers (1, 2, 3,…), fractions can also be added and subtracted. When counting fractions, we are counting the number of equally sliced pieces.

In this handout, we will focus on adding and subtracting proper fractions in two cases.
- **Case 1**: Denominators are the same
- **Case 2**: Denominators are different

Case 1: Denominators are the same
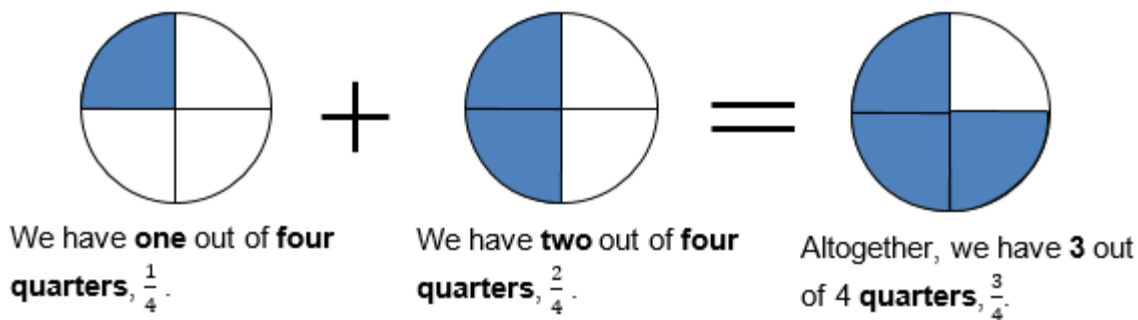
Step 1:

Keep the denominator the same.

Step 2:

Add or subtract the numerators.

Exercise 1: Add the fractions, $\frac{1}{4} + \frac{2}{4}$.

Let's draw a picture to see what this looks like.

The 4 in the **denominator** tells us that each whole circle is cut into **4 equal** portions. By adding the fractions, we are grouping the total number of pieces.

We have **one** out of **four** quarters, $\frac{1}{4}$.

We have **two** out of **four** quarters, $\frac{2}{4}$.

Altogether, we have **3** out of 4 **quarters**, $\frac{3}{4}$.

How does the math work?

Step 1: Since the two fractions have equal sized slices, keep the denominator the same, $\frac{?}{4}$.

Step 2: Add the numerators, $\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$.

Thus, we have $\frac{3}{4}$ of a whole.

Case 2: Denominators are different

Exercise 2: Add the fractions, $\frac{1}{4}$ and $\frac{1}{2}$.

Let's draw a picture to see what this looks like.

The 4 in the denominator of the first fraction tells us that the whole is cut into 4 equal slices. The 2 in the denominator of the second fraction tells us that the whole is cut into 2 equal slices.

We have **one** out of **four slices**, $\frac{1}{4}$.

We have **one** out of **two slices**, $\frac{1}{2}$.

Altogether, we have two slices that vary in size.

Can we add these two fractions together?

Since fractions are made up of equal sized slices, we can't have one slice smaller than another.

Note: We can only add or subtract fractions if their **denominators** are the **same.** If not, we must create equal sized slices by finding a **common denominator**.

Finding a Common Denominator

Step 1:

Find the Lowest Common Multiple (LCM) between the denominators of the given fractions.

Step 2:

Multiply the numerator and denominator of each fraction by a number so that they have the LCM as their new denominator. This process is called creating **equivalent fractions**.

Exercise 3: Add the two fractions, $\frac{1}{2} + \frac{1}{4}$.

Step 1: List the multiples of 2 and 4.
Multiplies of 2: 2, **4**, 6, 8, 10…
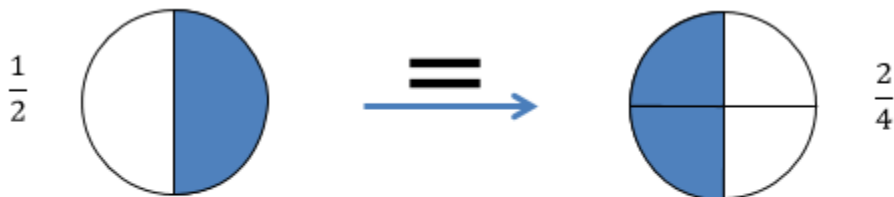**Multiplies of 4:** **4**, 8, 12, 16…

Note that 2 and 8 are multiples of 4 and 2 BUT 2 is the Lowest Common Multiple.

Step 2: a) We need to find a number that when multiplied to the top and bottom of $\frac{1}{2}$, we get the LCM (4) as the new denominator.
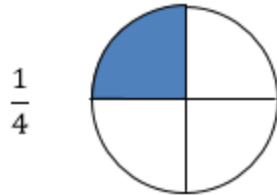
$$\frac{1 \times ?}{2 \times ?} = \frac{?}{4}$$

Since $2 \times 2 = 4$, we need to multiply the numerator and the denominator by **2**.

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$


$\frac{1}{2}$ = $\frac{2}{4}$

Notice that, $\frac{1}{2} = \frac{2}{4}$ since they share the same amount of shaded area. Hence, we created an **equivalent fraction**.

b) Looking at the second fraction, $\frac{1}{4}$, we notice that the LCM (4) is already in the denominator. Thus, we leave the fraction as it is.

$$\frac{1}{4}$$

Step 3: Now that our wholes are cut into equal sized slices, we can add their numerators just as we did in **Case 1**.

Thus, in total we have, $\frac{2}{4} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$ of a whole.

$$\frac{2}{4} \quad + \quad \frac{1}{4} \quad = \quad \frac{3}{4}$$
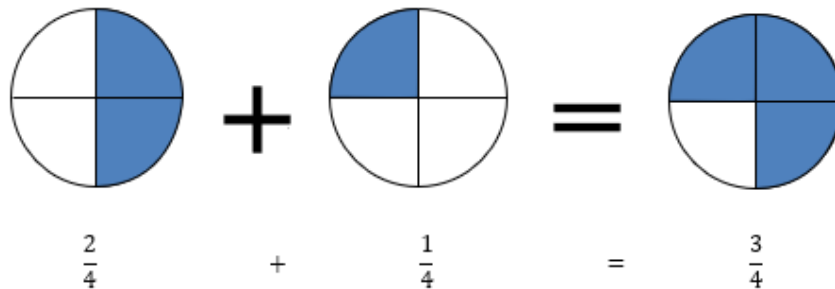
Exercise 4: Find a common denominator between the two fractions, $\frac{2}{3}$ and $\frac{1}{4}$ in order to create equivalent fractions.

Step 1: List the multiples of 3 and 4.
Multiplies of 3: 3, 6, 9, **12,** 15, 18, 21, **24**…
Multiplies of 4: 4, 8, **12,** 16, 20, **24,** 28…

Note that 12 and 24 are multiples of 3 and 4 BUT 12 is the **Lowest Common Multiple**.

Step 2: a) We need to find a number that when multiplied to the top and bottom of $\frac{2}{3}$, we get the LCM (12) as the new denominator.

$$\frac{2 \times ?}{3 \times ?} = \frac{?}{12}$$

Since $\times$ **4** = **12** , we need to multiply the numerator and the denominator by 4.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$\dfrac{2}{3}$ $\longrightarrow$ $\dfrac{8}{12}$

Notice that, $\dfrac{2}{3} = \dfrac{8}{12}$ since they share the same amount of shaded area.

$$\dfrac{1 \times ?}{4 \times ?} = \dfrac{?}{12}$$

Since $4 \times 3 = 12$, we need to multiply the numerator and the denominator by 3.

$$\dfrac{1 \times 3}{4 \times 3} = \dfrac{3}{12}$$

b) Now, we need to find a number that when multiplied to the top and bottom of $\dfrac{1}{4}$, we get the LCM (12) as the new denominator.
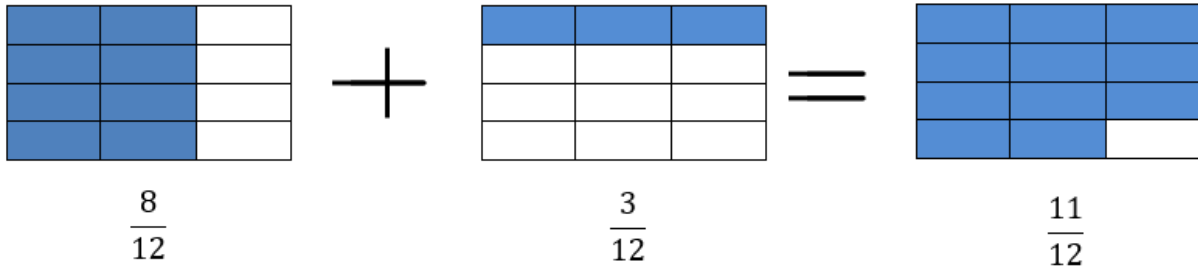

$\dfrac{1}{4}$ $\longrightarrow$ $\dfrac{3}{12}$

Notice that, $\dfrac{1}{4} = \dfrac{3}{12}$ since they share the same amount of shaded area.

Thus, our **equivalent fractions** are $\dfrac{2}{3} = \dfrac{8}{12}$ and $\dfrac{1}{4} = \dfrac{3}{12}$.

Exercise 5: Using our findings from Exercise 4, add the two fractions, $\dfrac{2}{3}$ and $\dfrac{1}{4}$.

In Exercise 3, we learned that $\dfrac{2}{3} = \dfrac{8}{12}$ and $\dfrac{1}{4} = \dfrac{3}{12}$.

Since our fractions now have equal sized slices, we can add their numerators just as we did in **Case 1**.

$$\frac{8}{12} \qquad \frac{3}{12} \qquad \frac{11}{12}$$

Thus, in total we have, $\frac{8}{12} + \frac{3}{12} = \frac{8+3}{12} = \frac{11}{12}$ of a whole.

Exercise 6: Perform subtraction on the following fractions, $\frac{1}{4} - \frac{1}{5}$.

Step 1: List the multiples of 4 and 5.

Multiplies of 4: 4, 8, 12, 16, **20**, 24, 28…
Multiplies of 5: 5, 10, 15, **20**, 25, 30…

The Lowest Common Multiple between 4 and 5 is **20**.

Step 2: a) We need to find a number that when multiplied to the top and bottom of $\frac{1}{4}$, we get the LCM (20) as the new denominator.

$$\frac{1 \times ?}{4 \times ?} = \frac{?}{20}$$

Since $4 \times \mathbf{5} = 20$, we need to multiply the numerator and the denominator by **5**.

$$\frac{1 \times \mathbf{5}}{4 \times \mathbf{5}} = \frac{\mathbf{5}}{\mathbf{20}}$$

Thus, $\frac{5}{20}$ is equivalent to $\frac{1}{4}$.

b) We need to find a number that when multiplied to the top and bottom of $\frac{1}{5}$, we get the LCM (20) as the new denominator.

$$\frac{1 \times ?}{5 \times ?} = \frac{?}{20}$$

Since $5 \times \mathbf{4} = 20$, we need to multiply the numerator and the denominator by 4.

$$\frac{1 \times \mathbf{4}}{5 \times \mathbf{4}} = \frac{\mathbf{4}}{\mathbf{20}}$$

Thus, $\frac{1}{5}$ is equivalent to $\frac{4}{20}$.

Step 3: Since our fractions now have equal sized slices, we can subtract their numerators just as we did in **Case 1**.

Thus, we now have,

$$\frac{5}{20} - \frac{4}{20} = \frac{5-4}{20} = \frac{1}{20}$$

of a whole.

Exercises:

1. Identify the LCD in the following pair of fractions.

a) $\frac{1}{3}, \frac{1}{6}$ b) $\frac{1}{3}, 1$ c) $\frac{4}{5}, \frac{2}{3}$ d) $\frac{3}{7}, \frac{2}{4}$

2. Add or subtract the following fractions. Remember to reduce.

a) $\frac{1}{2} + \frac{1}{2}$ b) $\frac{2}{4} - \frac{1}{4}$ c) $\frac{1}{5} + \frac{3}{5}$ d) $\frac{5}{2} + \frac{1}{2}$

e) $\frac{5}{8} + \frac{3}{8} + \frac{1}{8}$ f) $\frac{1}{1} - \frac{1}{1}$ g) $\frac{5}{8} - \frac{3}{8}$ h) $\frac{5}{5} - \frac{3}{5} + \frac{2}{5}$

3. Add or subtract the following fractions with different denominators. Remember to reduce.

a) $\frac{1}{3} + \frac{1}{6}$ b) $\frac{2}{5} + 2$ c) $\frac{1}{7} + \frac{2}{3}$ d) $\frac{5}{6} - \frac{1}{4}$

e) $\frac{3}{4} + \frac{4}{20}$ f) $\frac{4}{12} - \frac{1}{3}$ g) $\frac{4}{8} + \frac{1}{2}$ h) $\frac{7}{11} - \frac{5}{8}$

4. The advisor at the Tutoring Learning Centre is having an end of the year celebration. She decides to make cupcakes using ¾ cup of flour, ½ cup of cocoa power, ¼ cup of table salt, ½ cup of baking powder. In total, how cups of ingredients did she use?

Solutions:

1. a) **6**   b) **3**   c) **15**   d) **28**

2. a) **1**   b) $\frac{1}{4}$ c) $\frac{4}{5}$ d) **3**   e) $1\frac{1}{8}$ f) **0**   g) $\frac{1}{4}$ h) $\frac{4}{5}$

3. a) $\frac{1}{2}$ b) $2\frac{2}{5}$ c) $\frac{17}{21}$ d) $\frac{7}{12}$ e) $\frac{19}{20}$ f) **0**   g) **1**   h) $\frac{1}{88}$

4. **2 cups**