Adding and Subtracting Mixed Numbers and Improper Fractions

Just like our counting numbers (1, 2, 3,…), fractions can also be added and subtracted. When counting **improper fractions** and **mixed numbers**, we are counting the number wholes and parts.
Note: The rules for adding and subtracting improper fractions are the same as working with proper fractions.

Case 1: Adding and Subtracting Improper Fractions with Common Denominators

Step 1:

Keep the denominator the same.

Step 2:

Add or subtract the numerators.

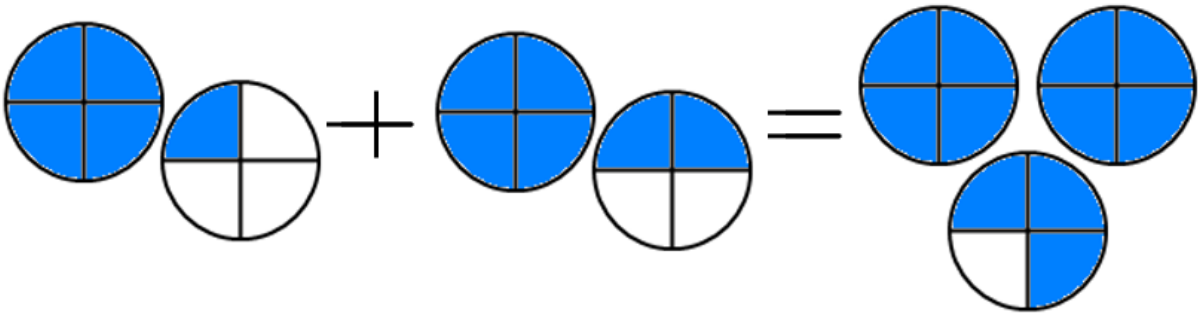
Step 3:

If the answer is an improper form, reduce the fraction into a mixed number.

Exercise 1: Add the fractions, $\frac{5}{4} + \frac{6}{4}$

Let's draw a picture to see what this looks like.

The 4 in the **denominator** tells us that each whole is cut into **4 equal** portions. By adding the fractions we are grouping the total number of wholes and parts.



We have **5** slices and each whole is made up of **4** slices, $\frac{5}{4}$.

We have **6** slices and each whole is made up of **4** slices, $\frac{6}{4}$

Altogether, we have **2 wholes** and 3 **quarters**, $2\frac{3}{4}$

How does the math work?

Step 1: Since the two fractions have equal sized slices, keep the denominator the same, $\frac{?}{4}$

Step 2: Add the numerators, $\frac{5}{4}+\frac{6}{4}=\frac{5+6}{4}=\frac{11}{4}$

Step 3: Thus, we have $2\frac{3}{4}$ wholes.

Case 2: Adding and Subtracting Improper Fractions with Different Denominators

Step 1:

Find the Lowest Common Multiple (LCM) between the denominators.

Step 2:

Multiply the numerator and denominator of each fraction by a number so that they have the LCM as their new denominator.

Step 3:

Add or subtract the numerators and keep the denominator the same.

Step 4:

If the answer is an improper form, reduce the fraction into a mixed number.

Exercise 2: Subtract the fractions, $\frac{7}{6} - \frac{3}{8}$

Step 1: List the multiples of 6 and 8.
Multiplies of 6: 6, 12, 18, **24**, 30, 36, 48…
Multiplies of 8: 8, 16, **24**, 32, 40, 48, 56…

The Lowest Common Multiple between 6 and 8 is **24**.

Step 2: a) We need to find a number that when multiplied to the top and bottom of $\frac{7}{6}$, we get the LCM (24) as the new denominator.

$$\frac{7 \times ?}{6 \times ?} = \frac{?}{24}$$

Since 6 x 4 = 24, we need to multiply the numerator and the denominator by **4.**

$$\frac{7 \times \mathbf{4}}{6 \times \mathbf{4}} = \frac{\mathbf{28}}{\mathbf{24}}$$

Thus, $\frac{7}{6}$ is equivalent to $\frac{28}{24}$.

b) We need to find a number that when multiplied to the top and bottom of $\frac{3}{8}$, we get the LCM (24) as the new denominator.

$$\frac{3 \times ?}{8 \times ?} = \frac{?}{24}$$

Since 8 x 3 = 24, we need to multiply the numerator and the denominator by **3.**

$$\frac{3 \times \mathbf{3}}{8 \times \mathbf{3}} = \frac{\mathbf{9}}{\mathbf{24}}$$

Thus, $\frac{3}{8}$ is equivalent to $\frac{9}{24}$.

Step 3: Since our fractions now have equal sized slices, we can subtract their numerators. Thus, we now have, $\frac{28}{24} - \frac{9}{24} = \frac{19}{24}$ of a whole.

Case 3: Adding and Subtracting Mixed Numbers Method 1

Step 1:

Convert all mixed numbers into improper fractions.

Step 2:

Check! Do they have a common denominator? If not, find a common denominator.

Step 3:

When necessary, create equivalent fractions.

Step 4:

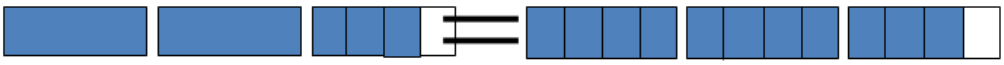
Add or subtract the numerators and keep the denominator the same.

Step 5:

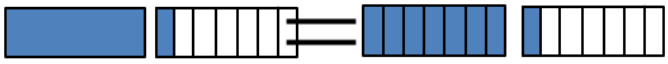
If the answer is an improper form, reduce the fraction into a mixed number.

Exercise 3: Subtract the fractions, $2\frac{3}{4} - 1\frac{1}{7}$

Step1: Convert both mixed numbers into improper fractions.



$$2\frac{3}{4} = \frac{(2\times 4)+3}{4} = \frac{\mathbf{11}}{\mathbf{4}}$$



$$1\frac{1}{7} = \frac{(1\times 7)+1}{7} = \frac{\mathbf{8}}{\mathbf{7}}$$

Step 2:

List the multiples of 4 and 7.

Multiplies of 4: 4, 8, 12, 16, 20, 24, **28**…
Multiplies of 7: 7, 14, 21, **28**, 35…

The Lowest Common Multiple between 4 and 7 is **28**.

Step 3: a) We need to find a number that when multiplied to the top and bottom of $\frac{11}{4}$ we get the LCM (28) as the new denominator.

$$\frac{11 \times ?}{4 \times ?} = \frac{?}{28}$$

Since 4 x 7 = 28, we need to multiply the numerator and the denominator by **7.**

$$\frac{11 \times \mathbf{7}}{4 \times \mathbf{7}} = \frac{\mathbf{77}}{\mathbf{28}}$$

Thus, $\frac{11}{4}$ is equivalent to $\frac{77}{28}$.

b) We need to find a number that when multiplied to the top and bottom of $\frac{8}{7}$, we get the LCM (28) as the new denominator.

$$\frac{8 \times ?}{7 \times ?} = \frac{?}{28}$$

Since 7 x 4 = 28 we need to multiply the numerator and the denominator by **4.**

$$\frac{8 \times \mathbf{4}}{7 \times \mathbf{4}} = \frac{\mathbf{32}}{\mathbf{28}}$$

Thus, $\frac{8}{7}$ is equivalent to $\frac{32}{28}$.

Step 4: Since our fractions now have equal sized slices, we can subtract their numerators. Subtracting their numerators we have $\frac{77}{28} - \frac{32}{28} = \frac{45}{28}$ of a whole.

Step 5: Thus, we have $1\frac{17}{28}$ wholes

Case 4: Adding and Subtracting Mixed Numbers Method 2

In this second method, we will break the mixed number into *wholes* and *parts*.

Step 1:

Add or subtract the *whole number part*.

Step 2:

Check! Does the *fraction part* share a common denominator? If not, find one.

Step 3:

When necessary, create equivalent fractions.

Step 4:

Add or subtract the numerators of the *fraction part* and keep the denominator the same.

Step 5:

If the answer is an improper form, reduce the fraction into a mixed number.

Exercise 4: Jessica is $19\frac{1}{2}$ years old today. How old was she $2\frac{1}{4}$ years ago?

Since we are looking at the difference between her current and past ages, our equation will look like: $19\frac{1}{2} - 2\frac{1}{4} = ?$

Step 1: Subtract the **whole number part**, 19 – 2 = 17.

Step 2: List the multiples of 2 and 4.

Multiplies of 2: 2, **4**, 6, 8…
Multiplies of 4: 4, 8, 12…

The Lowest Common Multiple between 2 and 4 is **4**.

Step 3: a) We need to find a number that when multiplied to the top and bottom of ½ we get the LCM (4) as the new denominator.

Since $2 \times \mathbf{2} = 4$, we need to multiply the numerator and the denominator by **2.**

$$\frac{1 \times \mathbf{2}}{2 \times \mathbf{2}} = \frac{\mathbf{2}}{\mathbf{4}}$$

Thus, $^1/_2$ is equivalent to $^2/_4$

b) Since $^2/_4$ already has the LCM (4) as the denominator, we leave the fraction as it is.

Step 4: Since our **fraction part** now has equal sized slices, we can subtract their numerators. Subtracting their numerators we have $\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$ of a whole.

Step 5: Combining our **whole number** and **fraction** parts we get,
17 wholes and $1/4$ = $17\frac{1}{4}$

Exercise 5: Subtract the fractions, $3\frac{1}{4} - 1\frac{3}{4}$

Step 1: Subtracting the **whole number part**, we get 3 – 1 = 2 wholes.

Step 2: Subtracting the **fraction part**, we get $\frac{1}{4} - \frac{3}{4} = ?$ of a whole.

Since we cannot take 3 away from 1, we need to ***borrow*** a whole from the first fraction.

Given $1/4$, let's ***borrow*** a whole by following the steps below:

Step 1. Rewrite *3 wholes* into *2 wholes + 1 whole*.

$$(2+1) \times \frac{1}{4}$$

Step 2. Since each whole has 4 slices, add the four slices from the borrowed whole into the numerator of the fraction part.

$$2\,\frac{1+\mathbf{4}}{\mathbf{4}}$$

We have created an equivalent fraction where $3\frac{1}{4}$ = $2\frac{5}{4}$

Step 3. Now we are able to subtract the fractions,

$$2\,\frac{5}{4} - 1\frac{3}{4}$$

Subtracting the **whole number part**, we are left with, 2 – 1 = 1 whole.

Subtracting the **fraction part**, we are left with

$$\frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$$

Combining our **whole number** and **fraction** parts we are left with,

1 whole and $\frac{1}{2} = 1\frac{1}{2}$

Exercises:

1) Add or subtract the following improper fractions and mixed numbers. Remember to reduce where possible.

a) $6\frac{5}{8} - 4\frac{3}{8}$ b) $6\frac{3}{8} + 9\frac{1}{24}$ c) $9\frac{9}{10} + 6\frac{7}{10}$ d) $\frac{10}{7} + \frac{11}{7}$

e) $\frac{9}{5} + \frac{14}{7}$ f) $1\frac{2}{4} - \frac{4}{3}$ g) $\frac{11}{8} + 3\frac{2}{3}$ h) $3 - \frac{6}{5}$

i) $1\frac{3}{5} - 1\frac{4}{9}$ j) $4\frac{1}{7} + 2\frac{1}{3} - \frac{3}{4}$ k) $5\frac{4}{5} + 8\frac{1}{3} - \frac{23}{4}$ l) $5\frac{4}{7} - 4\frac{6}{7}$

2. Each week Fred works $3\frac{1}{2}$ hours on Monday, 3 hours on Tuesday, 2 hours on Wednesday, $2\frac{1}{4}$ hours on Thursday, and 4 hours on Friday. How many hours does he work per week?
3. During a workshop, the English Tutors ate $3\frac{1}{2}$ pizzas and the Math Tutors ate $5\frac{2}{3}$ pizzas. How many pizzas were ordered? (**Hint**: Pizzas are ordered in wholes.)
4. The fourth floor of the D building has $600\frac{1}{2}$ ft^2 of space to house the TLC (Tutoring Learning Centre), SLC Student Learning Centre), and PAL (Peer Assisted Learning). If the TLC uses $120\frac{1}{4}$ ft^2 and the PAL uses $115\frac{1}{3}$ ft^2, how much space does SLC use?
5. It takes $2\frac{2}{3}$ hours to travel to Toronto from Waterloo while travelling with the GO. However, driving takes $1\frac{1}{8}$ hours. How much time do you save by driving?

Solutions:

1. a) $2\frac{1}{4}$ b) $15\frac{5}{12}$ c) $16\frac{3}{5}$ d) 3

e) $3\frac{4}{5}$ f) $\frac{1}{6}$ g) $5\frac{1}{24}$ h) $1\frac{4}{5}$

i) $\frac{7}{45}$ j) $5\frac{61}{84}$ k) $8\frac{23}{60}$ l) $\frac{5}{7}$

2. $14\frac{3}{4}$

3. 10 pizzas

4. $364\frac{11}{12}$

5. $1\frac{13}{24}$